UNIFORM DISTRIBUTION

1. A random variable x is uniformly distributed with mean 1 and variance $\frac{4}{3}$. Find $P(X<0)$.

Solution :

For uniform distribution

$$\text{Mean} = \frac{a+b}{2}$$

Given Mean $= 1$

$$\frac{a+b}{2} = 1$$

$$a+b = 2 \quad — \quad (1)$$

$$\text{Variance} = \frac{(a-b)^2}{12}$$

$$\frac{(a-b)^2}{\cancel{12}\,4} = \frac{4}{\cancel{3}}$$

$$(a-b)^2 = 16$$

$$a-b = 4 \quad — \quad (2)$$

Solving 1 & 2

$$2a = 6$$

$$\boxed{a = 3}$$

$$\boxed{b = -1}$$

The pdf of UD is $f(x) = \frac{1}{b-a} \quad a<x<b$

$$f(x) = \frac{1}{3+1} \qquad -1 < x < 3$$

$$f(x) = \frac{1}{4}, \quad -1 < x < 3$$

$$P(x<0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-1}^{0} \frac{1}{4}\,dx = \frac{1}{4}(x)_{-1}^{0}$$

$$\boxed{P(x<0) = \frac{1}{4}}$$

2. A random variable X has a uniform distribution over $(-3, 3)$. Compute $P(X<2)$, $P(|X|<2)$, $P(|X-2|<2)$. Find k for which $P(X>k) = \frac{1}{3}$.

Solution:

The pdf of UD is $f(x) = \frac{1}{b-a} \quad a<x<b$

$$f(x) = \frac{1}{3+3} \qquad -3 < x < 3$$

$$f(x) = \frac{1}{6}, \quad -3 < x < 3$$

(i) $$P(x<2) = \int_{-\infty}^{2} f(x)\,dx = \int_{-3}^{2} \frac{1}{6}\,dx = \frac{1}{6}[x]_{-3}^{2}$$

$$= \frac{1}{6}[2-(-3)]$$

$$\boxed{P(x<2) = \frac{5}{6}}$$

(ii) $$P[|x|<2] = P[-2<x<2] = \int_{-2}^{2} f(x)\,dx$$

$$= \int_{-2}^{2} \frac{1}{6}\,dx = \frac{1}{6}[x]_{-2}^{2} = \frac{4}{6}$$

(iii) $P[|X-2|<2] = P[-2 < X-2 < 2]$

$$= P[-2+2 < X-2+2 < 2+2]$$

$$= P[0 < X < 4]$$

$$= \int_0^4 f(x)\,dx$$

$$= \int_0^3 \frac{1}{6}\,dx = \frac{1}{6}[x]_0^3$$

$$= \frac{3}{6}$$

$$\boxed{P[|X-2|<2] = \frac{1}{2}}$$

(iv) To find $P[X>K] = \frac{1}{3}$

$$P[K<X] = \frac{1}{3}$$

$$P[K<X<3] = \frac{1}{3}$$

$$\int_K^3 f(x)\,dx = \frac{1}{3}$$

$$\int_K^3 \frac{1}{6}\,dx = \frac{1}{3}$$

$$\frac{1}{\cancel{6}_{\,2}}(x)_K^3 = \frac{1}{\cancel{3}}$$

$$\frac{1}{2}(3-K) = 1$$

$$3-K = 2$$

$$\boxed{K = 1}$$

3. Buses arrive at a specific stop at 15 minutes intervals starting at 7am. That is they arrive at 7, 7.15, 7.30, 7.45 and so on. If a passenger arrives at the stop at the time of uniformly distributed between 7 and 7.30 am. Find the probability that he waits (i) less than 5 minutes for a bus (ii) more than 10 minutes for a bus.

Solution:

X is a random variable which is uniformly distributed in the interval (0, 30).

$$f(x) = \begin{cases} \frac{1}{30} & 0 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P\{$Passengers will have to wait less than 5 mins$\}$ = $P\{$Passengers may arrive between 7.10 to 7.15$\}$ + $P\{$Passengers may arrive between 7.25 to 7.30$\}$

$$= \int_{10}^{15} \frac{1}{30}\,dx + \int_{25}^{30} \frac{1}{30}\,dx$$

$$= \frac{1}{30}\left[(x)_{10}^{15} + (x)_{25}^{30} \right]$$

$$= \frac{1}{30}\left[5+5 \right] = \frac{10}{30}$$

$$= \frac{1}{3}$$

(ii) P{ Passengers will have to wait atleast 12 mins } = P{ Passengers arrive b/w 7 to 7.3 } + P{ Passengers arrive b/w 7.15 to 7.18 }

$$= \int_0^3 \frac{1}{30}\,dx + \int_{15}^{18} \frac{1}{30}\,dx$$

$$= \frac{1}{30}\left[(x)_0^3 + (x)_{15}^{18}\right]$$

$$= \frac{1}{30}\left[3+3\right]$$

$$= \frac{6}{30}$$

$$= \frac{1}{5}$$